

Fig. 6 Shift of typical fluctuating-pressure spectrum with air-speed; possible adjustments of frequency/level detection windows.

angle of attack from 7° – 10° increases the pressure level in the 100-Hz frequency band by almost 30 db and that a change from 10° – 12° decreases the level in the 1000-Hz band by about 20 db.

It appears that a stall-warning system need not necessarily use the FPL spectrum in the entire audio-frequency range. One may generally obtain positive identification of a spectrum at a given air speed by observing the fluctuating-pressure signal in only two one-third octave bands.

In the design of a warning system for stall at a wide range of speeds, the effects of flow speed are also an important consideration. In scaling the FPL spectra, one takes the flow speed into account by referring the fluctuating pressure to the freestream dynamic pressure $q = \rho U_\infty^2/2$ and referring the frequency to U_∞/l (where l represents a characteristic length dimension). Figure 6 indicates how a spectrum may be expected to change with airspeed and also shows a number of possibilities as to how one might shift a level/frequency detection window of a stall-warning system to account for these speed-associated spectrum shifts.

Thus, detection of incipient stall by means of microphones flush-mounted at properly selected locations on the upper surfaces of aircraft wings appears feasible. The greatest and most easily detectable changes in the fluctuating-pressure spectrum sensed by a microphone occur as the flow reattachment line passes over the microphone; therefore, microphones located where reattachment occurs at a critical angle of attack (at a given speed) are particularly well-suited for detecting when the aircraft reaches that critical angle.

In designing a stall-detection system, one must also ensure that acoustic noise, such as results primarily from the propulsion system, does not mask the fluctuating-pressure signal used as the basis for stall detection. In many aircraft, the fluctuating-pressure spectrum at reattachment greatly exceeds that due to noise and thus the problem of masking is resolved; however, where this condition does not exist, additional signal processing (e.g., narrowband filtering, correlation, or averaging of signals from two sensors) may be used to extract the desired signal.

References

- Heller, H. H., Bliss, D. B., Ungar, E. E., and Widnall, S. E., "Feasibility of Aircraft Stall Detection by Means of Pressure Fluctuation Measurements," AFFDL-TR-70-147, Bolt, Beranack and Newman, Cambridge, Mass., Nov. 1970.

Turbulent Coaxial Jet Mixing in a Constant Area Duct

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Nomenclature

- P_T = total pressure
 r = radial coordinate
 r_e = radius of external duct
 r_j = radius of inner jet
 T_T = total temperature
 u = axial velocity component
 χ = axial coordinate
 ρ = density
 ϵ = eddy viscosity
 μ = turbulent viscosity $\mu = \rho \epsilon$
 $\bar{\mu}$ = nondimensionalized turbulent viscosity $\bar{\mu} = \mu/\rho_j u_j r_j$
 $()_e$ = initial external stream conditions
 $()_j$ = initial inner jet stream conditions
 $()_L$ = properties on the centerline
 $()$ = nondimensionalized by respective jet values

Introduction

THE mixing process which takes place when two moving streams come into contact is a flow problem which has application to many areas of fluid mechanics. For example, such mixing occurs in combustion chambers, jet pumps, and various propulsion systems. This present study is concerned with the turbulent mixing between two compressible, axisymmetric, coaxial jets confined in a constant area duct.

The flowfield in the duct consists of two distinct mixing regions: 1) a potential core region in which the flow properties are constant along the centerline, and mixing occurs only in the radial direction; and 2) a main mixing region in which flow properties change along the centerline and in the radial direction. This flowfield is analogous to that of a freejet mixing system in which the outer stream is infinite.

The freejet mixing process is fairly well understood. Many experiments have been verified by solutions of the turbulent boundary-layer equations utilizing the turbulent viscosity concept. Most of the experimental data for free mixing and the various models used in representing turbulent transport are summarized and discussed by Schetz¹ and Harsha.²

On the other hand the process of ducted coaxial jet mixing is not very well understood. In this study experimental centerline and radial distributions of total pressure are compared with theoretical predictions for compressible air/air mixing. Good agreement exists between theory and experiment provided the proper value of the turbulent viscosity is used in the calculations.

Theory

The analytical approach taken in this study is an explicit finite difference solution³ of the turbulent boundary-layer equations. The theory directly yields axial and radial distributions of velocity and static temperature throughout the flowfield. The static pressure is assumed to be a function only of the axial coordinate, and is obtained by an iteration procedure which is based on the total mass flow rate through the duct. With the velocity, temperature, and static pressure known, the total pressure is obtained by assuming locally isentropic flow. The boundary layer, which builds up on the external duct wall, is approximated by an incompressible flat plate model.

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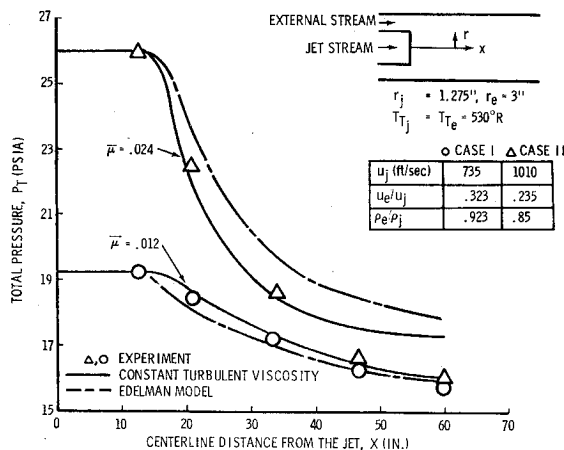


Fig. 1 Total pressure distributions along the centerline of the duct.

In the theoretical solution it is assumed that the turbulent viscosity is known a priori. In this study Ferri's⁴ model is used in the potential core region

$$\bar{\mu} = \kappa \bar{x} ((\rho u)_e - 1) \quad (1)$$

where κ is a constant.

In the main mixing region two different models are used. The first assumes that the turbulent viscosity is constant

$$\bar{\mu} = \text{const} \quad (2)$$

The second is the model employed by Edelman⁵:

$$\bar{\mu} = 0.018 \bar{r}_{1/2} (\bar{\rho} \bar{u})_L \quad (3)$$

where $\bar{r}_{1/2}$ is the nondimensionalized half-radius defined by the location of the mean mass flow rate per unit area across the duct. It is also assumed that the turbulent Prandtl and Schmidt numbers are equal to 1.

Comparison of Theory and Experiment

Experimental centerline distributions of total pressure for air/air jet mixing obtained by John⁶ in a constant area duct are compared with theoretical calculations in Fig. 1. The initial conditions in the jet and external stream are also presented in the figure. For both cases the constant in the potential core viscosity model was evaluated using a freejet mixing analysis. For the first case $\kappa = 0.0008$, and for the second $\kappa = 0.0005$. Also, the length of the potential core was taken from the experiment. For the first case ($u_j = 735$ fps) $\bar{\mu} = 0.012$ in the main mixing region results in a theoretical distribution which is in good agreement with the experimental distribution. Use of the Edelman model also results in good

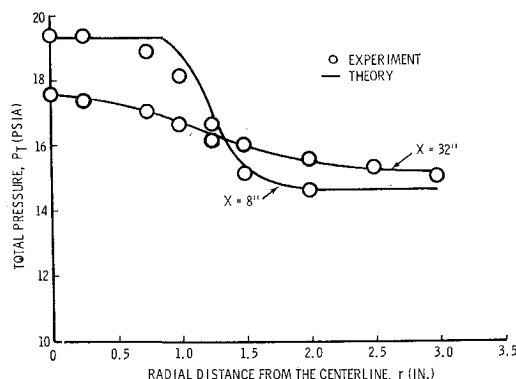


Fig. 2 Radial distributions of the total pressure, case I.

agreement between theory and experiment. In the second case ($u_j = 1010$ fps) $\bar{\mu} = 0.024$ gives good agreement between theory and experiment. For this latter case the Edelman model predicts too slow of a mixing process.

Experimental radial distributions of total pressure are compared with theoretical calculations in Fig. 2 for case I. The theory utilized Eqs (1) and (2). The agreement between theory and experiment is good for both the potential core and main mixing regions.

Certainly the good agreement between theory and experiment demonstrates the ability to predict coaxial jet mixing in a constant area duct. However, it is felt that extensive data analysis is needed to determine a reliable turbulent viscosity model for this type of mixing process.

References

- Schetz, J., "Analysis of the Mixing and Combustion of Gaseous and Particle-Laden Jets in an Airstream," AIAA Paper 69-33, New York, 1969.
- Harsha, T., "Free Turbulent Mixing: A Critical Evaluation of Theory and Experiment," AEDC-TR-71-36, Feb. 1971. Arnold Engineering Development Center, Arnold Air Force Base, Tenn.
- Baronti, P. and Rotta, N., "An Introduction to Mixing and Combustion Inside Channels," ATL-TR-126, Dec. 1968, Jericho, N.Y.
- Ferri, A., Libby, P., and Zakkay, V., "Theoretical and Experimental Investigation of Supersonic Combustion," Rept. 713, Sept. 1962, Polytechnic Inst. of Brooklyn, Brooklyn, N.Y.
- Edelman, R., and Fortune, O., "An Analysis of Mixing and Combustion in Ducted Flows," AIAA Paper 68-114, New York, 1968.
- John J., "An Experimental Investigation of the Bounded Mixing of Two Compressible Axially Symmetric Jet Streams," Aero Engineering Lab. Rept. 399, June 1957, Princeton Univ., Princeton, N.J.

Interpolation Using Surface Splines

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I. Introduction

A SURFACE spline is a mathematical tool for interpolating a function of two variables. It is based upon the small deflection equation of an infinite plate. The method was originally developed for interpolating wing deflections and computing slopes for aeroelastic calculations. The main advantages of the surface spline are that the coordinates of the known points need not be located in a rectangular array and the function may be differentiated to find slopes.

A linear spline, which is based upon the small deflection equation of an infinite beam, has been quite useful for one-dimensional interpolation. A lattice of linear splines has been used¹ to solve the two-dimensional problem. An advantage of the surface spline method it is that does not require the user to locate the splines.

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